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Technical Document 1453

September 1988

Quality Factor and Statistics for Radiation Patterns of Shipboard HF Antennas

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ADMINISTRATIVE INFORMATION

The following work was performed by the Naval Ocean Systems Center, Antenna and RF Systems Integration Branch, Code 822, under the technical direction of the Space and Naval Warfare Systems Command (SPAWAR), PMW 152-33D. Funding was provided by SPAWAR Work Request No. N0003988WXDL660 dated 24 May 1988. This work is closely related to FY 87 efforts performed under Naval Sea Systems Command (PMS-400) Work Request No. N0002407WR10102 dated 9 October 1986 and FY 88 efforts under Work Request No. N0002408WX10102 dated 23 September 1987.

Released by
J. B. Rhode, Head
Exterior Communications
Systems Branch

Under authority of
G. E. Ereckson, Head
Shipboard Systems Division

REPORT DOCUMENTATION PAGE

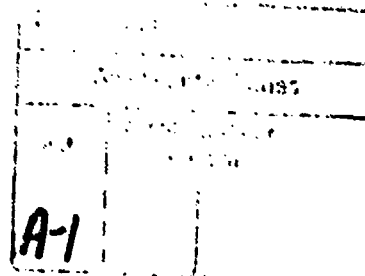
1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED			1b. RESTRICTIVE MARKINGS	
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION/AVAILABILITY OF REPORT	
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE			Approved for public release; distribution is unlimited.	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) NOSC Technical Document 1453			5. MONITORING ORGANIZATION REPORT NUMBER(S)	
6a. NAME OF PERFORMING ORGANIZATION Naval Ocean Systems Center		6b. OFFICE SYMBOL (if applicable) Code 824	7a. NAME OF MONITORING ORGANIZATION	
6c. ADDRESS (City, State and ZIP Code) San Diego, CA 92152-5000			7b. ADDRESS (City, State and ZIP Code)	
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Space and Naval Warfare Systems Command		8b. OFFICE SYMBOL (if applicable) PMW 152-33D	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	
8c. ADDRESS (City, State and ZIP Code) Ship Communications Systems Design Washington, DC 20363-5100			10. SOURCE OF FUNDING NUMBERS	
			PROGRAM ELEMENT NO. OMN	PROJECT NO. SPWR
			TASK NO. 825-CM02	AGENCY ACCESSION NO. DN912 045
11. TITLE (include Security Classification) QUALITY FACTOR AND STATISTICS FOR RADIATION PATTERNS OF SHIPBOARD HF ANTENNAS				
12. PERSONAL AUTHOR(S) E. A. Thowless, S. T. Li				
13a. TYPE OF REPORT Final		13b. TIME COVERED FROM Feb 1987 TO Sep 1988		14. DATE OF REPORT (Year, Month, Day) September 1988
15. PAGE COUNT 32				
16. SUPPLEMENTARY NOTATION				
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB-GROUP		
			HF communications, radiation pattern statistics	
19. ABSTRACT (Continue on reverse if necessary and identify by block number) The characteristics of various statistical entities of radiation patterns are reviewed in an effort to: improve the technical representation and definition of a figure-of-merit ("quality factor") for antenna patterns; improve understanding of the distribution of pattern gain values; determine requirements for antenna pattern statistical data for various applications; develop an improved "antenna pattern statistical summary" that employs the fewest numbers to provide a statistical characterization of radiation patterns.				
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT <input type="checkbox"/> UNCLASSIFIED/UNLIMITED <input checked="" type="checkbox"/> SAME AS RPT <input type="checkbox"/> OTHER			21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED	
22a. NAME OF RESPONSIBLE INDIVIDUAL E. A. Thowless			22b. TELEPHONE (include Area Code) (619) 553-3786	
			22c. OFFICE SYMBOL Code 824	

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

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1. OBJECTIVES

The objectives of this study were to:

- Review the characteristics of various statistical entities of radiation patterns,
- Improve the technical representation and definition of a figure of merit or "quality factor" for antenna patterns,
- Develop an improved understanding of the distribution of pattern gain values,
- Determine the need for antenna pattern statistical data for the antenna design engineer, for sponsors, for historical record purposes, and for link analysis applications,
- Attempt to keep the statistical characterization of radiation patterns simple (with the fewest numbers) and meaningful for the antenna design engineer and for the communications systems engineer,
- Develop in connection with these goals, an improved "antenna pattern statistical summary" which is a record of each measured antenna.

2. BACKGROUND

This report is an update on the title subject (see references below). Several aspects of pattern statistics as used in the past are presented in improved or modified form; for example, quality factor and standard deviation.

S.T. Li, Memorandum: "Pattern Range Output," ser 8112/45, NOSC, 4 May 1981.

E.A. Thowless, Memorandum: "Definitions for Radiation Pattern Statistics," ser 8122 95-81, NOSC, 31 Dec 1981.

J.W. Rockway, Memorandum: "Pattern Range Output," ser 811/311 NOSC, 29 May 1977.

"Statistical Analysis of Antenna Radiation Patterns, Final Report," (unpublished) NOSC, 9 Sep 1983, written by Mike Naber but not so indicated on the document.

3. RADIATION PATTERN OF COMBINED POLARIZATIONS

The ionosphere is an anisotropic medium that alters (with very few exceptions) the polarization of any hf sky wave signal refracted from the ionosphere. Because of this behavior, knowledge of the individual component polarizations (i.e., vertical or horizontal) of an antenna is of limited value, other than requiring vertical polarization for ground wave propagation. A radiation pattern representing the total power radiated is more meaningful than patterns individually representing E_v (vertical) polarization and E_h (horizontal) polarization.

The field gain of the total power pattern, in a given spatial direction, is derived from data points of vertical and horizontal radiation patterns. This field gain for space wave (sky wave) propagation is:

$$g_t = (|g_v|^2 + |g_h|^2)^{1/2} \quad (1)$$

where

$$\begin{aligned} |g_\theta|^2 &= \text{power equivalent of the data point of gain for vertical polarization} \\ &= \log^{-1} (G_\theta / 10) , \end{aligned} \quad (1a)$$

where G_θ is the measured dB value of gain for vertical polarization and

$$\begin{aligned} |g_\phi|^2 &= \text{power equivalent of the data point of gain for horizontal polarization} \\ &= \log^{-1} (G_\phi / 10) , \end{aligned} \quad (1b)$$

where G_ϕ is the measured dB value of gain for horizontal polarization.

$$\text{The dB equivalent of } g_j \text{ is: } G_j = 20 \log (g_j) . \quad (2)$$

The gain reference for the measured patterns at the Antenna Pattern Range (APR) is the maximum amplitude (near 0 deg elevation) measured for a quarter wavelength vertical monopole over a good conducting ground plane at the frequencies of calibration. For the antenna pattern statistical summary, the gain reference will remain as described above. For quality factor and for link analysis applications, gain relative to an isotropic radiator is used. These two gain references are related as:

$$G_{dBi} = G_{dBq} + 5.161 \text{ dB} . \quad (3)$$

For ground wave pattern statistics, only the vertical polarization patterns at 5 deg (or lower) elevation angle are used. The ground wave field gain for a data point from these patterns is:

$$g_{gwa} = |g_\theta| = \log^{-1} (G_\theta / 20) . \quad (4)$$

4. QUALITY FACTOR (QF)

Quality factor in current and past usage describes only the circularity of the conical-cut patterns — how well each pattern radiates uniformly toward all azimuths. Note that a nearly perfectly circular pattern can have a very low gain and still be tagged with a very high quality factor. This should not be.

A new proposed quality factor includes the circular quality, and a gain quality factor that also compensates for (excludes) the antenna impedance mismatch loss at the frequency of measurement. The new proposed quality factor is conceptually expressed as:

$$QF = (\text{CIRCULAR QUALITY}) * (\text{GAIN QUALITY}) = CQ * GQ .$$

Quality factor is determined on the basis of the distribution of RF power in the hemisphere. Consequently, all gain values employed for quality factor (circular and gain qualities) are power gains. Power gain is represented by symbol g in Section 4 of this report. Elsewhere in the report, g represents field (voltage) gain.

4.1 CIRCULAR QUALITY

Circular quality is a measure of the amount of pattern power gain that falls below the average power gain, expressed such that 1.0 represents a perfect circular pattern, and with decreasing values representing worsening irregularity of the pattern shape. The circular quality of a single conical-cut pattern at a given elevation angle, ψ , is:

$$\text{Circular Quality} = CQ = 1 - \frac{1}{N} \sum_{j=1}^N \left[1 - \left(\frac{g_j}{\bar{g}_\psi} \right)_{<1} \right], \quad (6)$$

where

N = number of measured data points of gain, typically 360 for a conical-cut pattern (uniformly spaced every degree in azimuth)

j = j^{th} measured data point

g_j = power gain of the j^{th} data point relative to isotropic

$$= \log^{-1} (G_j / I_0) \quad (6A)$$

\bar{g}_ψ = mean (average) power gain of the conical-cut pattern (at elevation angle ψ)

$$= \frac{1}{N} \sum_{j=1}^N g_j \quad (6B)$$

$$\left(\frac{g_j}{\bar{g}_\psi} \right)_{<1} = \text{only power gain } (g_j) \text{ values less than the mean gain of the conical-cut pattern } (\bar{g}_\psi) \text{ are considered for circular quality (i.e., the ratio is less than 1.0).}$$

4.2 GAIN QUALITY RATIO, UNCORRECTED FOR MISMATCH LOSS

Occasionally the average gain of a conical-cut pattern is very low. This can occur when the elevation angle of a conical-cut pattern coincides with a null (more noticeable in a vertical-plane pattern) due to vertical-plane lobing. This type of null can result from current phase reversals on the antenna, commonly occurring when an antenna structure becomes greater than a half wavelength.

The vertical-plane radiation pattern of a quarter-wave vertical monopole serves as the gain reference for determining the gain quality of conical-cut patterns at all measured elevation angles. To account for the effects of pattern gain on quality factor, the mean power gain of a conical-cut pattern measured at a given elevation angle is compared to the power gain of an ideal quarter wave monopole at the same elevation angle. This comparison is called the gain quality ratio (Eq. 7).

Thus gain quality ratio, uncorrected for mismatch loss, is given as:

$$GQ_{uc} = \frac{g_\psi}{g_{q\psi}}, \quad (7)$$

where

\bar{g}_ψ = mean power gain of the conical-cut pattern at elevation angle ψ ,

$g_{q\psi}$ = pattern power gain of an ideal quarter-wave reference monopole at the same elevation angle, ψ .

The radiation pattern of a hemispherically isotropic radiator conceivably could be used for a reference. However, the quarter-wave monopole was chosen because: (1) most all ship-board hf antennas are monopoles or variants thereof, (2) of the need to have more gain at lower elevation angles than at higher angles for ground wave and long range sky wave propagation, and (3) there is little need to radiate as much power overhead as toward the horizon.

When the need arises to consider the quality of pattern coverage for near-vertical incidence, then admittedly the null at zenith, where the reference monopole gain is zero, becomes a problem. When measured near vertical incidence, pattern information becomes commonplace; then this overhead area may be treated in a special manner for quality factor determination. It is not addressed at this time.

4.3 IMPEDANCE MISMATCH CONSIDERATIONS

The 35-ft whips, commonly used as receiving antennas, have very large impedance mismatch losses at some frequencies, especially at the lower hf frequencies. These losses normally do not cause significant degradation of the receiving system's signal-to-noise ratio because of the high levels of atmospheric or man-made noise (though they are considered for link analysis).

Transmitting antennas, if serious mismatch losses exist, are used with fixed impedance matching networks or antenna tuners. In either case, antenna mismatch loss does not impact quality factor, and its effect upon antenna gain is removed.

The relative amount of power reflected (not radiated) is the power reflection coefficient, ρ^2 , which is related to the standing-wave-ratio (SWR) as presented in Eq. (8).

$$\text{power reflection coefficient} = \rho^2 = \left(\frac{\text{SWR} - 1}{\text{SWR} + 1} \right)^2, \quad (8)$$

Antennas on the 1:48-scale ship models are measured without impedance matching devices of any kind. Any impedance mismatch loss will manifest itself as a decrease in measured antenna gain. The effect of mismatch loss upon measured antenna gain of the model antennas is compensated for in order to achieve a more accurate gain quality value.

Two ways exist to determine mismatch loss. The conventional method is from impedance measurements, typically done when the ship model is at the "impedance pit" for antenna impedance and coupling measurements. The other method is to calculate the mismatch loss from a set of conical-cut radiation patterns for a given frequency.

4.4 MISMATCH LOSS DERIVED FROM RADIATION PATTERNS

SWR and impedance mismatch loss can be calculated from a set of conical-cut patterns. The requirement is that the set of patterns adequately accounts for (or represents) the distribution of the radiated power in space.

The concept derives from the following logic and assumptions:

- (1) The average power gain at each measured elevation angle is determinable from the respective conical-cut pattern.
- (2) The amount of area of the hemisphere represented by each conical-cut pattern is determinable. The assumption made is that the statistics of the pattern are representative of the radiation throughout the limited horizontal band of area that extends above and below the elevation angle of the pattern (see Figure 1).
- (3) The product of the average pattern gain of a conical-cut pattern and the relative area represented by each pattern is equal to the amount of rf power within that area relative to the total radiated power.
- (4) The sum of these gain-area products (a total of six in the examples that follow) is the total rf power radiated. The sum should be 1.0. (Stated in another manner: all the power being radiated is accounted for by the radiation patterns).
- (5) Usually, however, the sum of the gain-area products will be less than 1.0, meaning a loss of some kind exists.
- (6) The antenna models and the brass ship models are virtually loss-less (no resistive losses).
- (7) The power not accounted for (in (5) above) is power not being radiated. Assuming no resistive losses associated with the antenna or its immediate environment, the loss must therefore be the reflected power due to the impedance mismatch loss of the antenna under test.
- (8) Being more specific, at the coax-antenna interface, the following condition exists:

$$\text{power reflected} = \text{power incident} - \text{power transmitted}, \quad (9)$$

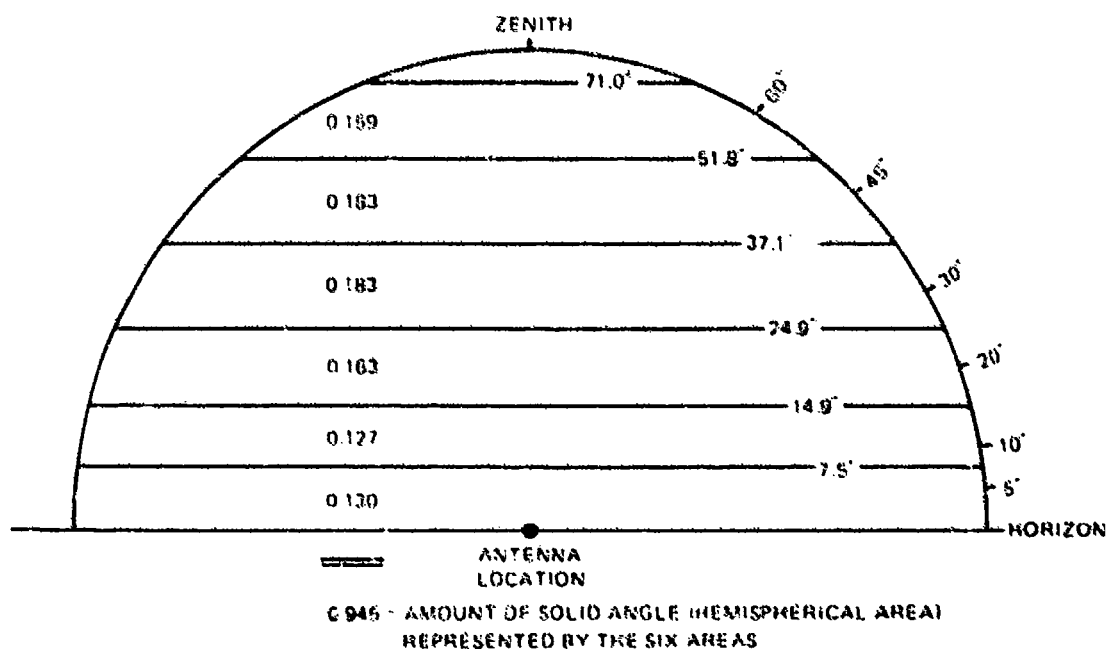


Figure 1. Side-view representation of hemisphere for pattern measurements, showing (1) elevation angles as currently used of conical-cut patterns (5°, 10°, 20°, etc.), (2) elevation angles between the conical-cut pattern angles where the areas between measurement angles are divided equally (7.5°, 14.9°, etc.), and (3) the relative area represented by each conical-cut pattern (0.130, 0.127, etc.).

or

$$P_{\text{refl}} = P_{\text{inc}} - P_t \quad (10)$$

(9) Other relationships:

$$\text{Transmitted power} = P_t = (1 - \rho^2) P_{\text{inc}} \quad (11)$$

$$\text{Mismatch loss} = \frac{P_t}{P_{\text{inc}}} = 1 - \frac{P_{\text{refl}}}{P_{\text{inc}}} = 1 - \rho^2, \quad (12)$$

where

ρ^2 = power reflection coefficient

ρ = voltage reflection coefficient.

Now we will look at the areas of the hemisphere represented by the conical-cut patterns. Figure 1 portrays a side view of the hemisphere of antenna radiation pattern measurements (since radiation patterns are measured at a fixed, though arbitrary, far-field distance). Table 1 lists the items and values to be discussed.

Table 1. Antenna radiation pattern parameters.				
Elevation Angle of Conical-Cut Pattern	Relative Area Between 0° and ψ	Elevation Angle That Halves the Area Between Adjacent Conical-Cut Angles	Relative Amount of Area of Hemisphere Represented by Conical-Cut Pattern	
ψ	$\sin \psi$	$\sin^{-1} \left(\frac{\sin \psi_{i+1} + \sin \psi_i}{2} \right)$	$\frac{1}{2}[(\sin \psi_{i+1} + \sin \psi_i) - (\sin \psi_i + \sin \psi_{i-1})] =$ (Except upper & lower areas)	K_0
Equation - to - use:				
5°	0.0872	7.49°	$\frac{1}{2}(\sin 5^\circ + \sin 10^\circ)$	0.1304
10°	0.1736	14.04°	$\frac{1}{2}(\sin 20^\circ + \sin 5^\circ)$	0.1374
20°	0.3420	24.90°	$\frac{1}{2}(\sin 30^\circ + \sin 10^\circ)$	0.1632
30°	0.5000	37.12°	$\frac{1}{2}(\sin 45^\circ + \sin 20^\circ)$	0.1825
45°	0.7071	51.87°	$\frac{1}{2}(\sin 60^\circ + \sin 30^\circ)$	0.1830
60°	0.8660	70.99°*	$\frac{1}{2}(\sin 60^\circ + \sin 45^\circ)$	0.1589
*The uppermost angle, the one above 60°, is given by:				$K = 0.9454$
$\sin^{-1} \left(\frac{\sin \psi_i + \sin \psi_{i-1}}{2} + \sin \psi_i \right) = 70.99^\circ$				
Note: about 5% of the coverage ($1 - K = 1 - 0.9454 = 0.0546$) is not represented (the area above 70.99°) for this particular set of conical-cut patterns. For our purposes it is assumed that the radiation coverage and behavior above 70.99°, relative to the reference pattern of a quarter wave monopole, is comparable to what occurs below 70.99°. For 5% of the hemispherical coverage, such an assumption is of minimal risk.				

The six elevation angles currently used for patterns are marked in Figure 1 at 5, 10, 20, 30, 45, and 60 deg. This selection of angles is such that each represents approximately (very roughly) an equal area of the hemisphere. Or stated differently, if a hemispherically isotropic radiator were located at the "antenna location," then the sampling at each conical-cut pattern (angle) would be representative of roughly equal rf power. The relative amount of hemispherical area between 0 deg (horizon) and any elevation, ψ , is proportional to $\sin \psi$ (see Table 1, second column). The K_ψ values in Table 1 are the relative amounts of the area of the hemisphere represented by each of the six conical-cut patterns.

The horizontal lines of Figure 1 marked with degrees as 7.5, 14.9, etc., represent the angular distance (elevation angle) that divides the area between adjacent conical-cut angles into two equal areas. The assumption made for determining rf power within each band is that the radiation pattern and its statistics at a given conical-cut angle is representative of the area bounded by adjacent boundaries. For example, the radiation pattern at 30 deg is assumed to be representative of at all elevation angles between 24.9 and 37.1 deg. The 71-deg line is the upper limit in this example, and the area between 60.0 and 71.0 deg is identical to the area between 51.8 and 60.0 deg.

The angle, A , that divides the area between two adjacent conical-cut patterns into two equal areas is (see Table 1, third column):

$$A = \sin^{-1} \left(\frac{\sin \psi_{j+1} - \sin \psi_j}{2} \right), \quad (14)$$

where

ψ_j = elevation angle of lower conical-cut pattern

ψ_{j+1} = elevation angle of higher adjacent pattern.

The relative amount of area of the hemisphere represented by a given conical-cut pattern, at elevation angle, ψ_j , is therefore:

$$K_\psi = 1/2 [(\sin \psi_{j+1} - \sin \psi_j) + (\sin \psi_{j+2} - \sin \psi_{j+1})]. \quad (15)$$

Note that the sum of K_ψ values should be 1.00, but for this given set of conical-cut patterns the total accounts for 94.54% of the hemisphere (Table 1, fifth column). The approximately 5% not represented is the area above 71 deg. For our purposes for QF determination, it is assumed that above 71 deg the radiation coverage relative to the reference pattern of a quarter-wave monopole is similar (in a quality factor sense) as to what occurs below 71 deg. For 5% of the hemispheric coverage, in an area with normally little radiated power from vertical-monopole-type antennas, such an assumption is of minimal risk.

As a demonstration of the accounting for all radiated power, an example is given in Table 2. Also refer to Figure 1. The average power gain of each conical-cut pattern is listed in the second column of Table 2. The product of this average power gain and the relative amount of hemispheric coverage (third column) is shown in the last column. Summing the

values of the last column gives the relative amount of radiated power (0.760) for the six areas represented by the patterns (0.945, see Figure 1). The total radiated power is obtained by accounting for the "non-represented" area (5%) by the ratio:

$$\frac{P}{K} = \frac{0.760}{0.945} = 0.804 = P_t, \quad (16)$$

where P_t is the power transmission coefficient, or the fraction of power transmitted across the antenna terminal-transmission line interface (because of mismatch loss).

Table 2. An accounting of all radiated power for an example set of mean power gains from a set of patterns.			
Elevation of Conical-Cut Pattern ψ	Mean Power Gain of Patterns (Example) g_ψ	Relative Hemispherical Area Coverage K_ψ	Relative Power Represented by each Conical-Cut Pattern $P_\psi = g_\psi K_\psi$
5°	1.35	0.130	0.176
10°	1.24	0.127	0.157
20°	0.942	0.163	0.154
30°	0.583	0.183	0.186
45°	0.431	0.183	0.079
60°	0.556	0.159	0.088
		$K = 0.945$	$P = 0.760$
To account for the approximate 5% of non-coverage, divide 0.760 by 0.945 to obtain the assumed total amount of radiated power for the whole hemisphere relative to incident power (=1.0):			
$\frac{P}{K} = \frac{0.760}{0.945} = 0.804 = P_t$			

The mismatch loss, for this example is:

$$L_m = -10 \log (1 - p^2) = -10 \log (0.804) = 0.947 \text{ dB}, \quad (17)$$

which represents a SWR of 2.59.

4.5 GAIN QUALITY

Having determined P_t , gain quality independent of mismatch loss is now calculatable. The average power gain is, in effect, increased to account for impedance mismatch by dividing the measured/calculated average gain g_ψ by the coefficient of transmission, P_t . This ratio relative to the gain of a quarter-wave monopole at the same elevation angle, ψ , is the gain quality factor for use for QF:

$$\text{Gain Quality} = GQ = \left(\frac{g_\psi / P_t}{g_{q\psi}} \right) < 1 \quad (18)$$

4.6 QUALITY FACTOR EQUATION

Quality factor, being the product of circular quality [Eq.(6)] and gain quality [Eq. (18)], is finally defined for a given conical-cut elevation angle, ψ , as:

$$Q^F = \left\{ 1 - \frac{1}{N} \sum_{j=1}^N \left[1 - \left(\frac{g_j}{\bar{g}_\psi} \right)_{<1} \right] \right\} \times \left(\frac{\bar{g}_\psi / P_t}{g_{q\psi}} \right)_{<1} \quad (19)$$

where

N = number of data points, typically 360 for a single pattern

g_j = power gain of the j^{th} data point

\bar{g}_ψ = mean power gain for the conical-cut pattern at elevation angle, ψ

$()_{<1}$ = use only resultant values within the parentheses that are less than 1

P_t = power transmission coefficient

$g_{q\psi}$ = power gain of a quarter-wave monopole at elevation angle, ψ .

4.7 APPARENT SWR

Knowing P_t allows the calculation of other related characteristics, all repeated or summarized below.

$$P_t = \text{power transmission coefficient} = (1 - \rho^2) P_{\text{inc}}, \quad \text{where } P_{\text{inc}} \text{ is assumed to be unity.} \quad (20)$$

$$L_m = \text{mismatch loss in dB} = -10 \log (1 - \rho^2). \quad (21)$$

$$\rho^2 = \text{power reflection coefficient} = 1 - P_t. \quad (22)$$

$$\rho = \text{voltage reflection coefficient}. \quad (23)$$

$$\text{SWR} = (1 + \rho) / (1 - \rho). \quad (24)$$

SWR calculated in this manner from P_t will be called "apparent SWR" to distinguish it from more conventional direct measurement of SWR. The accuracy of apparent SWR determined from patterns has not as yet been estimated. Overall accuracy will be a function of: (1) the calibration accuracy and procedure for the reference quarter-wave monopoles, (2) the tolerances of the APR equipment, (3) the electrical stability of all the rf components and equipment, both active and passive, employed for pattern measurements, and (4) the soundness of the assumption that a conical-cut pattern is a reasonable representation of pattern behavior within the brief range of elevation angles above and below.

If apparent SWR is determined for a set of conical-cut patterns of a reference quarter-wave monopole, the result should be 1.0. Any great variation from 1.0 will indicate an instrumentation problem. Very small departures should be indicative only of equipment tolerances. Large deviations warrant investigations for the causes.

5. COMPARISONS AMONG MEAN, STANDARD DEVIATION, COEFFICIENT OF VARIATION, AND GAIN REFERENCES FOR PATTERN DATA

Assume that from a set of dB gain values, mean gain, standard deviation, and coefficient are determined. Then convert the resultant dB values of the three statistical characteristics to power or field values. These power or field values of the statistical characteristics will differ from the mean gain, standard deviation, and coefficient of variation obtained directly from a column of power gain values, or a column of field gain values, where both the set of power gain values and the set of field gain values were derived from the original set of dB gain values.

The first column of Table 3 is an abbreviated set of gain data from a measured radiation pattern. The gain reference for this example is the maximum gain of a quarter-wave monopole. The units are dB, and for this gain reference is designated as dB_q. The second and third columns are the equivalent power (PWR_q) and field (voltage) (FLD_q) gains: $PWR_q = \log^{-1} (dB_q/10)$; $FLD_q = \log^{-1} (dB_q/20)$.

The fourth through sixth columns are the same gain data but in units of dBi, power (PWR_i), and field (FLD_i) gain values relative to an isotropic radiator, where

$$G_{dBi} = G_{dBq} + 5.161 \text{ dB} \quad (24)$$

Any of the six columns could arguably be considered the appropriate set of values for pattern gain use, and for the calculation of statistical quantities. Let us look at the statistical quantities of mean, standard deviation, and coefficient of variation [see Eq. (29)] as derived from the six columns of differing sets of numbers, each set representing the same radiation pattern, though in differing units.

The statistical values at the bottom of Table 3 without parentheses are derived from the column of gain values directly above. The values in parentheses are obtained from the numbers without parentheses. For example, in Column 2, 0.698 is the mean power gain, and derived from that is the -1.561-dB gain and the .835-field gain. Similarly, in Column 4, 5.922 is the dB standard deviation, while 3.910 and 1.977 are the power and field standard deviations, respectively, derived from 5.922 dB.

A careful review of the three sets of rows of statistics, at the bottom of the table, labeled mean, standard deviation, and coefficient of variation, yields the following observations and conclusions:

1. Mean gains relative to isotropic are 5.161 dB greater than the mean gains relative to quarter-wave monopole. This difference is the gain difference between the two reference antennas [as shown in Eq. (24)], or 3.282 as a power ratio, and 1.812 as a voltage ratio. Nothing unusual here.
2. Standard deviations for the two columns of dB value, dB_q and dBi, are identical. This is because the values in the two columns differ by a constant difference value (5.161).
3. Standard deviations for the two columns of power values differ by a ratio of 3.282, and standard deviations for the two columns of field values differ by a ratio of 1.812. Both these ratios are equivalent to 5.161 dB, the difference of the gain references.
4. Coefficients of variation derived from dB values are meaningless. For any set of values not bounded by zero (such as these dB values which straddle zero) the coefficient of variation is invalid. For such sets of numbers, when the value of the mean approaches zero, the coefficient of variation becomes increasingly larger, becoming infinite when the mean equals zero (regardless of the standard deviation).

Table 3. Comparisons among mean, standard deviation, and coefficient of variation for units of dB, power and field (volts), and for two gain references. All six columns represent the same radiation intensity from an antenna.

Gain Relative to Quarter-wave Monopole			Gain Relative to Isotropic Radiator		
dB _q	PWR _q	FLD _q	dB _i	PWR _i	FLD _i
-7.9	0.1622	0.4027	-2.739	0.5322	0.7295
-20.6	0.0087	0.0933	-15.439	0.0286	0.1691
-9.0	0.1259	0.3548	-3.839	0.4131	0.6428
-6.4	0.2291	0.4786	-1.239	0.7518	0.8671
-5.5	0.2818	0.5309	-0.339	0.9249	0.9617
-6.8	0.2089	0.4571	-1.639	0.6856	0.8280
-8.0	0.1585	0.3981	-2.839	0.5201	0.7212
-4.4	0.3631	0.6026	0.761	1.1915	1.0916
-11.8	0.0661	0.2570	-6.639	0.2168	0.4656
-4.9	0.3236	0.5689	0.261	1.0619	1.0305
-1.7	0.6761	0.8222	3.461	2.2187	1.4895
-6.6	0.2188	0.4677	-1.439	0.7180	0.8473
-0.9	0.8128	0.9016	4.261	2.6675	1.6332
1.1	1.2882	1.1350	6.261	4.2277	2.0561
3.3	2.1380	1.4622	8.461	7.0162	2.6488
4.0	2.5119	1.5849	9.161	8.2433	2.8711
3.3	2.1380	1.4622	8.461	7.0162	2.6488
-0.7	0.8511	0.9226	4.461	2.7932	1.6713
Mean =					
dB	-4.639	(-1.561)	0.522	(3.598)	(2.265)
pwr	(0.344)	0.698	(1.128)	2.290	(1.685)
fld	(0.586)	(0.835)	(1.062)	1.513	(1.298)
Std. dev. =					
dB	5.922	(-1.124)	5.922	(4.035)	(-2.192)
pwr	(3.910)	0.772	(3.910)	2.532	(0.604)
fld	(1.977)	(0.879)	(1.977)	(1.591)	(0.777)
Coef. of Var. =					
dB	-1.277*	(0.438)	11.342*	(0.438)	(-4.451)
pwr		1.106		1.106	0.359
fld		(1.052)		(1.052)	(0.599)

*meaningless numbers

5. Coefficients of variation for the two power columns are identical, and similarly for the two field columns.

6. The means derived from dB, power and field values are not transformable (equatable) from one to the other.

An example is presented. The 0.698 mean power gain relative to a quarter-wave monopole has an equivalent dB value of $10 \log (0.698) = -1.561$ dB, and an equivalent field value of $(0.698)^{1/2} = 0.835$. Neither of these values matches the values derived from the column of dB values (-4.639 dB) or from the column of field values (0.717).

7. Standard deviations of dB, power, and field values also are not transformable (equatable) from one to the other.

8. Coefficients of variation of dB, power, and field values are not transformable (equatable) from one to the other.

6. CURRENT ANTENNA PATTERN RANGE (APR) STATISTICS PRACTICE

Currently the antenna pattern statistical summary sheets have mean gains and "standard deviation" derived from the field values of pattern gains, but expressed in dB. Using Table 3 as an example (see third column), the mean (dB) and standard deviation (dB) as currently calculated and used at the APR are:

$$\text{mean} = 20 \log (0.717) = -2.89 \text{ dB}_q \quad (25)$$

$$\text{"std. dev."} = 20 \log (0.429) = -7.35 \text{ dB} \quad (26)$$

What does a "standard deviation" of $20 \log (0.429)$ really mean? What is the denominator of the ratio represented by 0.429? It is 1.0; but 1.0 what? A negative "standard deviation" is also confusing.

Also note what occurs when a different reference is used for pattern gain, say an isotropic radiator instead of a quarter-wave monopole. The values for mean and "standard deviation" are now (see Table 3, column 6)

$$\text{mean} = 20 \log (1.298) = 2.27 \text{ dBi}, \quad (27)$$

$$\text{"std. dev."} = 20 \log (0.777) = -2.19 \text{ dB}. \quad (28)$$

The difference between the two means (-2.89 and 2.27) is the difference between the two gain standards; that is expected. The difference in "standard deviations" (-7.351 and 2.192) is also the same as the difference in the gain standards. Standard deviation is supposed to represent the dispersion of a set of values relative to their mean. It is difficult to have a realistic understanding of the magnitude of dispersion of data if "standard deviation" can differ from -7.35 to -2.19 dB, depending on the gain reference used for pattern gain measurements.

"Standard deviation," as currently obtained, leads to confusion in understanding the results. It is obtained by first calculating the "standard deviation" of the field gain values (relative to quarter-wave monopole), and then converting that result into dB. "Standard deviation" of the field gains results in a value less than one. When expressed in dB, large negative dB values of "standard deviation" result from small dispersion of gain values, and small negative dB values result from large dispersion of gain values (see Table 3, third column, "standard deviation" values of 0.429 and 7.35 dB, as an example). "Standard deviation," properly used, has larger values representing larger dispersions of data and no negative numbers.

7. CASE FOR COEFFICIENT OF VARIATION

Coefficient of variation (V) can be effectively used as a replacement for standard deviation of a set of field gain values in order to present a more understandable quantification of data dispersion:

$$V = \frac{\sigma}{\bar{X}} = \frac{\text{std. dev.}}{\text{mean}} \quad (29)$$

Note that coefficient of variation is a unitless value. It more meaningfully portrays the extent of variability, because it gets away from the absolute units of standard deviation, accounts for the specific value of the mean, and compares the standard deviation relative to the mean — sort of a “normalized standard deviation.”

For the example discussed in Section 6, and for both sets of field values (for the two gain references):

$$V = \frac{\sigma}{\bar{g}} = \frac{0.429}{0.717} = \frac{0.777}{1.298} = 0.599 \quad (30)$$

The coefficient of variation shows the standard deviation for both sets of field gain data as 59.9% of the arithmetic mean, which is more understandable than the calculated values of standard deviation (0.429 and 0.777), which differ and convey little insight.

Coefficient of variation is proposed for use for the antenna pattern statistical summary sheets instead of the presently used “standard deviation” in dB.

8. STATISTICS FOR THE ANTENNA PATTERN STATISTICAL SUMMARY

Equations for pattern statistics for the antenna pattern statistical summary sheets, the antenna design engineer, and APR use are delineated in this section (statistics for communication link analysis are in Section 9). Antenna field gains for the antenna patterns statistical summary are referenced to the maximum measured gain of a quarter-wave monopole. Two sets of statistics are to be derived; one for ground wave only, and the other for space wave (sky wave). Except as noted, the equations presented below are used for both ground and space waves.

8.1 STATISTICAL CONSIDERATIONS FOR GROUND WAVE

HF ground wave is a common and important mode of rf propagation. Statistics for ground wave, which utilizes vertical polarization only, should not be distorted by data from horizontal polarization and by data from higher elevation angles. Only radiation patterns for vertical polarization at the lowest possible elevation angle are used (0 deg is preferable, but 5 deg is typically used at NOSC with negligible error). Block composites need not be used for ground wave statistics, since there is only a single contributing conical-cut pattern; hence, the values of means, coefficient, and standard deviation will not differ.

8.2 STATISTICAL CONSIDERATIONS FOR SPACE WAVE (SKY WAVE)

Whereas the statistics derived for ground wave pattern behavior are derived from a series of frequencies at only the lowest elevation angle, space wave pattern statistics encompass all the elevation angles measured (typically six) through a range of frequencies. Space wave pattern statistics are derived from field gain values of the total power patterns, consisting of the sum of the powers of the horizontal and vertical polarization patterns [see Eq. (1)].

8.3 THE STATISTICS EQUATIONS

8.3.1 Mean Gain (\bar{g} ; \bar{G})

The mean gain is derived from the field gain values for the conical-cut pattern of interest. The mean field gain is:

$$\bar{g} = \frac{1}{N} \sum_{j=1}^N g_j, \quad (31)$$

and the mean gain expressed in dB is:

$$G = 20 \log (\bar{g}), \quad (32)$$

where

j = j^{th} data point

N = number of measured data points, typically 360 (every degree in azimuth)

g_j = field gain of the j^{th} data point

8.3.2 Mean Gain, Block Composite (\bar{g}_{BC} ; \bar{G}_{BC})

The block composite consists of the typically six ($= M$) conical-cut patterns for space waves at a given frequency. Expressed as field gain, the block composite mean gain is

$$\bar{g}_{BC} = \sum_{j=1}^M \frac{N_j \bar{g}_j \cos \psi_j}{N_1 + N_2 + \dots + N_M}, \quad (33)$$

and expressed in dB, it is

$$\bar{G}_{BC} = 20 \log (\bar{g}_{BC}).$$

where

N_j = number of measured data points (typically 360) contributing to the mean, \bar{g}_j , of the j^{th} pattern

\bar{g}_j = mean gain for the j^{th} data point

M = number of patterns (elevation angles)

$(\cos \psi)$ = weighting factor used to compensate for decreasing areas as ψ increases

ψ_j = elevation angle of the j^{th} conical-cut pattern

For some antennas, such as the 2- to 6-MHz fan, which may also be used for receiving through a wider frequency range (2-12 MHz), two block composites may be used, one for transmit ($\bar{G}_{BC,tx}$), and the other for receive ($\bar{G}_{BC,rx}$).

8.3.3 Mean Gain, Overall Composite (\bar{g}_{OA} ; \bar{G}_{OA})

The overall composite includes the pattern gains for all the frequencies.

$$\bar{g}_{OA} = \sum_{j=1}^L \frac{N_j \bar{g}_j}{N_1 + N_2 + \dots + N_6}, \quad (34)$$

and expressed in dB, it is

$$\bar{G}_{OA} = 20 \log (\bar{g}_{OA}) \cdot \text{dB}_q.$$

where

N_j = number of measured data points contributing to the mean, \bar{g}_j , of the j^{th} frequency

\bar{g}_j = mean field gain for the j^{th} frequency; this is from block composites for sky waves, or mean gains for ground wave patterns

L = number of means (for frequencies) contributing to the overall composite.

8.3.4 Standard Deviation (σ_{BC})

Standard deviation, representing the dispersion of values of the data points making up the mean gain, is calculated for the purpose of determining the coefficient of variation, but is not printed out on any pattern statistical summary sheet. The standard deviation is

$$\sigma = \left[\frac{1}{N} \sum_{j=1}^N (g_j - \bar{g})^2 \right]^{1/2}, \quad (35)$$

where

g_j = field gain of the of the i^{th} data point

\bar{g} = mean field gain

N = number of measurd data points, typically 360

j = j^{th} data point.

8.3.5 Standard Deviation, Block Composite (σ_{BC})

This block composite standard deviation is calculated as a step in determining the block composite coefficient of variation and is not printed out on the pattern statistical summary sheet. The block composite value is obtained by iterating Eq. (36) for each conical-cut pattern, using respective σ , \bar{g} , N , and ψ values. The initial calculation includes the two sets of values of the first two patterns. The resultant σ , \bar{g} , and N is used in the next iteration, which will now also include the σ , \bar{g} , N , and ψ values representing the next conical-cut pattern. The σ resulting from the final iteration is the block composite standard deviation:

$$\sigma_{BC\ 1,2} = \left(\frac{N_1 \sigma_1^2 + N_2 \sigma_2^2}{N_1 + N_2} + \frac{N_1 N_2 (\bar{g}'_1 - \bar{g}'_2)^2}{(N_1 + N_2)^2} \right)^{1/2}, \quad (36)$$

where

N_1 & N_2 = number of measured data points for patterns 1 and 2, respectively (typically 360 for a single pattern)

σ_1 & σ_2 = standard deviations for patterns 1 and 2, respectively

\bar{g}'_1 & \bar{g}'_2 = $\bar{g}_1 \cos \psi_1$, and $\bar{g}_2 \cos \psi_2$ for patterns 1 and 2, respectively. $\cos \psi$ is ignored when \bar{g} (and associated σ and N) represents the results(s) of combining two or more sets of σ , \bar{g} , N , and ψ .

The values of field gain and of N to use for a subsequent iteration are given as:

$$\bar{g}_{1,2} = \frac{N_1 \bar{g}_1 + N_2 \bar{g}_2}{N_1 + N_2} \quad (37a)$$

$$N_{1,2} = N_1 + N_2. \quad (37b)$$

For example, assume $\sigma_{1,2}$ and $\sigma_{3,4}$ have been calculated for data sets 1 and 2, and 3 and 4, with mean gains of $\bar{g}_{1,2}$ and $\bar{g}_{3,4}$, respectively.

The block composite standard deviation for these four pattern data sets is:

$$\sigma_{BC\ 1,2,3,4} = \left(\frac{(N_1 + N_2) \sigma_{1,2}^2 + (N_3 + N_4) \sigma_{3,4}^2}{N_1 + N_2 + N_3 + N_4} + \frac{(N_1 + N_2) (N_3 + N_4) (\bar{g}_{1,2} - \bar{g}_{3,4})^2}{(N_1 + N_2 + N_3 + N_4)^2} \right)^{1/2}. \quad (38)$$

8.3.6 Standard Deviation, Overall Composite (σ_{OA})

The overall composite standard deviation for space waves is a combining of the block composite of the sky wave patterns. For ground waves the overall composite standard deviation consists of the standard deviations (treated as block composites) of the ground wave patterns. This σ_{OA} is not part of the pattern statistical summary sheet, but is used to determine

the overall composite coefficients of variation. The equations and manipulations are similar to Eq. (36), (37), and (38). Overall composite standard deviation for a combination of two (1 & 2) block composites is:

$$\sigma_{OA\ 1,2} = \left(\frac{M_1 \sigma_{BC1} + M_2 \sigma_{BC2}}{M_1 + M_2} + \frac{M_1 M_2 (\bar{g}_{BC1} - \bar{g}_{BC2})^2}{(M_1 + M_2)^2} \right)^{1/2}, \quad (39)$$

where

M_1 and M_2 = number of test points, which is the sum of all the test points of all the patterns contributing to block composite 1 (BC1), and to block composite 2 (BC2), respectively

σ_{BC1} and σ_{BC2} = the standard deviations of block composites one and two, respectively

\bar{g}_{BC1} and \bar{g}_{BC2} = the mean field gains of the data points of block composites one and two, respectively.

The mean field gain for the two block composites, \bar{g}_{BC1} and \bar{g}_{BC2} is :

$$\bar{g}_{OA\ 1,2} = \frac{M_1 \bar{g}_{BC1} + M_2 \bar{g}_{BC2}}{M_1 + M_2}. \quad (40)$$

And, for combining two composite combinations such as $\sigma_{OA\ 1,2}$ and $\sigma_{OA\ 3,4}$ for an overall composite, the equation is [like Eq. (38)]:

$$\sigma_{OA\ 1,2,3,4} = \left(\frac{(M_1 + M_2)\sigma_{OA\ 1,2}^2 + (M_3 + M_4)\sigma_{OA\ 3,4}^2}{M_1 + M_2 + M_3 + M_4} + \frac{(M_1 + M_2)(M_3 + M_4)(\bar{g}_{OA\ 1,2} - \bar{g}_{OA\ 3,4})^2}{(M_1 + M_2 + M_3 + M_4)^2} \right)^{1/2}. \quad (41)$$

For calculating standard deviation overall composites for ground waves, use the \bar{g} 's and σ 's (instead of \bar{g}_{BC} and σ_{BC}) from the ground wave patterns for all the measured frequencies.

8.3.7 Coefficient of Variation (V)

Coefficient of variation is like a normalized standard deviation. It is a unitless value expressing the amount of data dispersion for a given pattern deviation, (see Sections 5 and 6). For ground wave pattern statistics the equation is:

$$V = \frac{\sigma}{\bar{g}}, \quad (42)$$

where

σ = standard deviation [see Eq. (35)]

\bar{g} = mean field gain [see Eq. (31)].

8.3.8 Coefficient of Variation, Block Composite (V_{BC})

This block composite is the ratio of the block composite standard deviation relative to the corresponding block composite mean gain:

$$V_{BC} = \frac{\sigma_{BC}}{\bar{g}_{BC}}, \quad (43)$$

where

σ_{BC} = block composite standard deviation [see Eq. (36) and (38)]

\bar{g}_{BC} = block composite mean field gain (see Eq. 33).

8.3.9 Coefficient of Variation, Overall Composite (V_{OA})

This overall composite is the ratio of the overall composite standard deviation relative to the corresponding overall composite mean gain:

$$V_{OA} = \frac{\sigma_{OA}}{\bar{g}_{OA}}, \quad (44)$$

where

σ_{OA} = overall composite standard deviation [see Eq. (39) and (41)]

\bar{g}_{OA} = overall mean field gain [see Eq. (34)].

8.3.10 Quality Factor (QF)

QF is defined by Eq. (19) for a given conical-cut pattern. For ground wave it is calculated for only vertical polarization at the lowest elevation angle measured (typically 5 deg).

8.3.11 Quality Factor, Block Composite (QF_{BC})

The block composite for quality factor is simply the mean value of the contributing QF's at a given frequency. It is used for only sky wave patterns:

$$QF_{BC} = \frac{1}{M} \sum_{j=1}^M QF_j, \quad (45)$$

where

M = number of QF values contributing to the block composite, typically six, the number of elevation angles measured

QF_j = the j^{th} value of contributing quality factor.

8.3.12 Quality Factor, Overall Composite (QF_{OA})

The overall composite for quality factor is the mean of the contributing QF's at a given frequency. For sky waves, the contributing QF's are the block composites. For ground waves, the contributing QF's are the ground wave QF's determined at each frequency.

$$QF_{OA} = \frac{1}{L} \sum_{j=1}^L QF_j, \quad (46)$$

where

L = number of QF values contributing to the overall composite for the various frequencies

QF_j = the jth value of contributing quality factor.

9. PATTERN STATISTICS FOR COMMUNICATION SYSTEMS ANALYSIS

All statistics for communication systems analysis utilize only pattern gain values referenced to an isotropic radiator and expressed in dBi. Except to determine the total power pattern [Eq. (1)], no field gain values are used for this set of pattern statistics discussed in this section. Equation (3) gives the gain relationship between an isotropic radiator and a quarter-wave monopole.

Two sets of pattern statistics are to be provided for each frequency of measurement: (1) ground wave only and (2) space wave. For ground wave pattern statistics, the dBi values of the data points for only the vertical polarization pattern of the lowest measured elevation angle (5 deg or less) are used. For space wave pattern statistics, the dBi values of the combined power pattern [Eq. (1)] are used.

The statistics equations for ground-wave-only use, and for sky wave use are mostly identical. Where differences exist, they will be noted.

Each set of pattern statistics for link analysis includes the following:

- mean gain
- standard deviation
- median gain
- 5th and 95th percentiles
- 1st and 9th decile
- 1st and 3rd quartiles
- cumulative frequency diagram

9.1 MEAN GAIN

The mean gain equation is similar to Eq. (31), but uses dBi values in the summation:

$$\bar{G} = \frac{1}{N} \sum_{j=1}^N G_j, \quad (47)$$

where G_j = dBi gain of the j^{th} data point.

9.2 MEAN GAIN, BLOCK COMPOSITE

For space wave block composite mean gain, the six (typically) elevation angles for a single frequency contribute to the block composite. The gain at each elevation angle is weighted by the cosine of the elevation angle.

$$\bar{G}_{BC} = \frac{1}{M} \sum_{j=1}^M \bar{G}_{\psi_j} \cos \psi_j, \text{ dBi}, \quad (48)$$

where

\bar{G}_{ψ_j} = mean gain of the conical-cut pattern, dBi

$\cos \psi_j$ = weighting function to account for the elevation angle of pattern

j = j^{th} pattern

M = number of conical-cut patterns contributing to the block composite.

For some antennas, such as the 2- to 6-MHz fan which may also be used for receiving through a wider frequency range (2-12 MHz), two block composites may be used, one for transmit ($\bar{G}_{BC, TX}$), and the other for receive ($\bar{G}_{BC, RX}$).

9.3 MEAN GAIN, OVERALL COMPOSITE

The overall composite includes the pattern gains for all the frequencies.

$$\bar{G}_{OA} = \frac{1}{L} \sum_{j=1}^L \bar{G}_j, \quad (49)$$

where

\bar{G}_j = mean gain (dBi) for the j^{th} frequency. This is from block composites for sky wave patterns, or from mean gains for ground wave patterns

L = number of means (for frequencies) contributing to the overall composite.

9.4 STANDARD DEVIATION

The standard deviation for a single conical-cut pattern is:

$$\sigma = \left(\frac{1}{N} \sum_{j=1}^N (G_j - \bar{G})^2 \right)^{1/2}, \text{ dBi}, \quad (50)$$

where

G_j = dBi gain of the j^{th} data point

\bar{G} = mean gain of the conical-cut pattern [from Eq. (47)]

N = number of measured data points.

9.5 STANDARD DEVIATION, BLOCK COMPOSITE

The block composite represents the standard deviation for all the conical-cut patterns (elevation angles) at a given frequency. The calculation is an iteration process as described in 7.3.5 for Eq. (36). The equation for combining two sets of data, where each set consists of σ , \bar{G} , N , and ψ , is:

$$\sigma_{\text{BC } 1,2} = \left(\frac{N_1 \sigma_1^2 + N_2 \sigma_2^2}{N_1 + N_2} + \frac{N_1 N_2 (\bar{G}'_1 - \bar{G}'_2)^2}{(N_1 + N_2)^2} \right)^{1/2}, \quad (51)$$

where

N_1 & N_2 = number of measured data points for patterns 1 and 2, respectively

σ_1 & σ_2 = standard deviations for patterns 1 and 2, respectively

$\bar{G}'_1 = \bar{G}_1 \cos \psi_1$, and $\bar{G}'_2 = \bar{G}_2 \cos \psi_2$ for patterns 1 and 2, respectively. $\cos \psi$ is ignored when \bar{G} (and associated σ and N) represents the result(s) of combining two or more sets of σ , \bar{G} , N , and ψ . \bar{G}_1 & \bar{G}_2 are the dBi mean gains for patterns 1 and 2, respectively.

The values of gain and of N to use for a subsequent iteration are given as:

$$\bar{G}_{1,2} = \frac{N_1 \bar{G}_1 + N_2 \bar{G}_2}{N_1 + N_2}, \text{ dBi} \quad (52a)$$

$$N_{1,2} = N_1 + N_2. \quad (52b)$$

A similar mean gain, $\bar{G}_{3,4}$, is determined for patterns 3 and 4 having σ_3 , \bar{G}_3 , and σ_4 and \bar{G}_4 , respectively.

The block composite standard deviation of $\sigma_{\text{BC } 1,2}$ and $\sigma_{\text{BC } 3,4}$ that will provide a block composite for σ_1 , σ_2 , σ_3 , and σ_4 is:

$$\sigma_{BC\ 1,2,3,4} = \left(\frac{(N_1 + N_2)\sigma_{1,2}^2 + (N_3 + N_4)\sigma_{3,4}^2}{N_1 + N_2 + N_3 + N_4} + \frac{(N_1 + N_2)(N_3 + N_4)(\bar{G}_{1,2} - \bar{G}_{3,4})^2}{(N_1 + N_2 + N_3 + N_4)^2} \right)^{1/2} \quad (53)$$

9.6 STANDARD DEVIATION, OVERALL COMPOSITE

The overall composite standard deviation for space waves is a combination of the block composites of the sky wave patterns. For ground waves, the overall composite standard deviation consists of the standard deviations (treated as block composites) of the ground wave patterns.

The equations are the same as Eq. (39) through (41).

9.7 PERCENTILES: MEDIAN, QUARTILES, AND DECILES

Median, quartiles, and deciles are all subsets of percentiles. The median may be considered the "average of position." Of a set of numbers, the median is the value with the same number of test points larger in value as there are test points smaller in value.

Quartiles (there are three) divide the array into four parts such that each part contains the same number of test points. The second quartile is, of course, identical to the median (or the 50th percentile). The first quartile divides the smaller-value half of the array into equal parts, and the third quartile divides the larger-value half of the array into equal parts.

Deciles are the nine points that divide the array into 10 equally populated groups.

It is assumed that the computer can handle array sizes of large numbers of data points, and that grouping of data into classes will not be necessary.

Assume an array with N data points. The *positions* (not value) of the following percentiles within the array are:

$$\text{First quartile} = Q_1 = \frac{(N + 1)}{4}$$

$$\text{Median} = \text{Med.} = Q_2 = \frac{2(N + 1)}{4} = \text{second quartile} \\ = \text{fifth decile}$$

$$\text{Third quartile} = Q_3 = \frac{3(N + 1)}{4}$$

$$\text{First decile} = D_1 = \frac{(N + 1)}{10}$$

$$\text{Ninth decile} = D_9 = \frac{9(N + 1)}{10}$$

$$\text{Fifth percentile} = P_5 = \frac{5(N + 1)}{100}$$

$$\text{Ninty-Fifth percentile} = P_{95} = \frac{95(N + 1)}{100}$$

$$j^{\text{th}} \text{ percentile} = P_j = \frac{j(N + 1)}{100}$$

As stated, the results of the above equations are for the position within the array of the percentile of interest. Once the percentile position is known, the percentile value can be determined.

If the calculated result for a percentile position does not correspond exactly to a value in the array, an interpolation must be done to determine the correct value. For example, suppose an array size of $N = 360$, then $Q_3 = 3(N + 1)/4 = 270.75$. If the 270th value of the array were, for example, -1.6 dBi, and the 271st value were -1.0 dBi, then the value of the third quartile (Q_3) would be three-fourths $(270.75 - 270)$ the way from -1.6 toward -1.0 dBi, or:

$$-1.6 + [.75 [(-1.0) - (-1.6)]] = -1.6 + (.75 \times 0.6) = -1.15 \text{ dBi.}$$

9.8 CUMULATIVE FREQUENCY DIAGRAM

A graph of a cumulative frequency diagram, also known as a "more-than" ogive, gives an excellent graphical portrayal of the distribution of gain values.

Numerical values for this diagram are obtained by using the equations of the previous section. Obviously a graph of this type could be plotted first, and from it median, quartile, etc., values then visually determined.

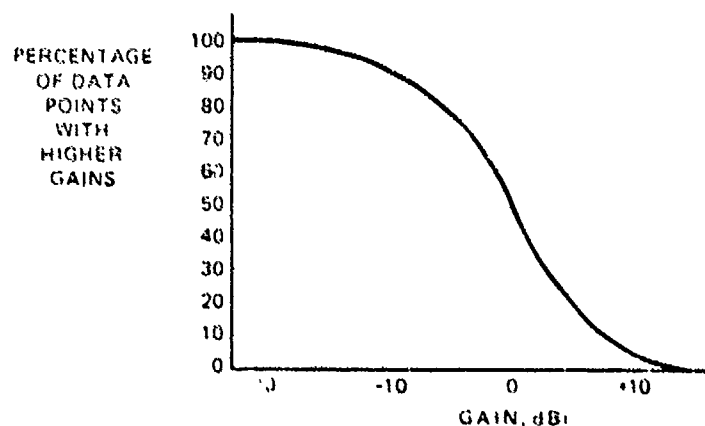


Figure 2. An example of a cumulative frequency diagram.

10. CONCLUSIONS

1. No single quantity can describe an antenna pattern, though the proposed quality factor may approach that objective. Besides reflecting pattern irregularity, QF values will now spread through a larger "dynamic range," reflect low overall pattern gains, and are not biased by antenna impedance mismatch loss.

2. For the Antenna Pattern Statistical Summary, mean gain derived from voltage (field) values and converted to dBq, coefficient of variation, and quality factor will be the recommended quantities for tabulation.

3. For communication systems analysis, quantities must be expressed in dBi, including standard deviation. The statistical quantities to be tabulated, all derived from dBi gain values, will be mean gain, standard deviation, median, first and third quartiles, the first and ninth decile, and the fifth and ninety-fifth percentiles. Cumulative frequency diagrams (graphs) will be printed for mean gains.

4. Though not discussed in the text, the gain values of a pattern do not have a normal distribution. When the means of power gains expressed in dB, field (voltage) and power are compared, the mean in dB has the lowest value, and the mean in power gain is the highest. The distribution of field gains is closest to being normal.

5. Statistics of mean, standard deviation, and coefficient of variation calculated using values of dB, field, or power are not transformable to either of the other units. For example, the mean calculated from dB values is not equal to the mean calculated in field (voltage) gain and then converted to dB.

6. The coefficient of variation, which could be considered a "normalized standard deviation," portrays unambiguously the amount of pattern dispersion. It is derived from field gain values.

7. It is the opinion of the authors (1) that all pattern statistics should be derived from field gain values; (2) that median and percentile values may represent the way to proceed in the future, primarily because they are convertible among dB, field, and power units without ambiguity; and (3) that median value is more meaningful than the mean when working with skewed distributions.

8. Space wave statistics have overall composite values encompassing a set of measured pattern data for azimuth angles, elevation angles, frequencies, and both vertical and horizontal polarizations. Ground wave statistics have overall composite values encompassing a set of measured pattern data for only vertical polarization at 5 deg (or less) elevation angle, azimuth angles, and frequencies (higher elevation angles and horizontal polarization are excluded).

11. RECOMMENDATIONS

1. It is recommended that the statistical entities discussed herein be incorporated into the next version of the pattern data program for use at the APR for use by antenna design engineers, in the antenna pattern statistical summary, and for communication system link analysis.

2. It has not escaped our attention that this treatment of pattern statistics is still an interim step. The major effort in the future will be to achieve a single set of statistics usable by both the ship's antenna design engineer, and the link analysis engineer.

3. A CG 47 class ship should be selected from which to measure more than the usual number of radiation patterns from a ship model. The principal objective is to ensure that the derived pattern statistics become a useful and understandable instrument that reflect the real world. Toward this end, various combinations of data will be evaluated, the presentation of the data will be improved, and modifications made where necessary.